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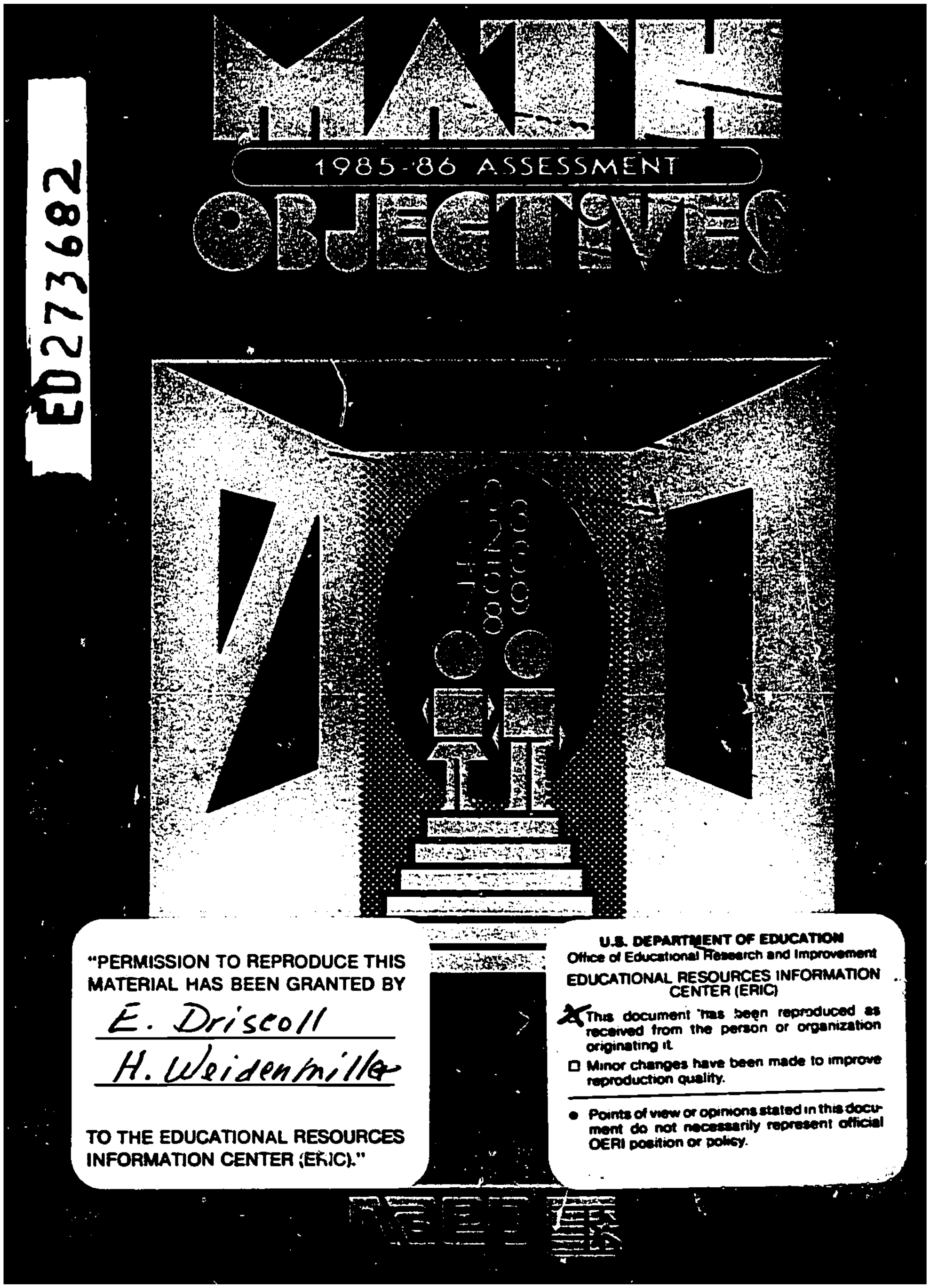
ABSTRACT

The National Assessment of Educational Progress (NAEP) collected data on the mathematics performance of 9-, 13-, and 17-year old Americans in 1973, 1978, and 1982. An additional assessment in grades 3, 7, and 11 is planned for 1986. Educational objectives to be used in the 1986 assessment were contributed, reviewed, and revised by a committee of educators. Emphasis was placed on higher-level, critical thinking skills. The objectives were grouped into the following categories, which are described in this document: (1) processes-routine application; understanding; problem solving and reasoning; skills; and knowledge; (2) content-fundamental methods; discreet mathematics; data organization and interpretation; measurement; geometry; relations; functions; and algebraic expressions; and numbers and objectives; and (3) attitudes--mathematics in school; mathematics and oneself; mathematics and society; mathematics as a discipline; and attitudes toward computers. Objectives in five categories concerning the use of calculators are also illustrated: routine computations; more difficult computations; understanding concepts; exploration; and applications and problem solving. (GDC)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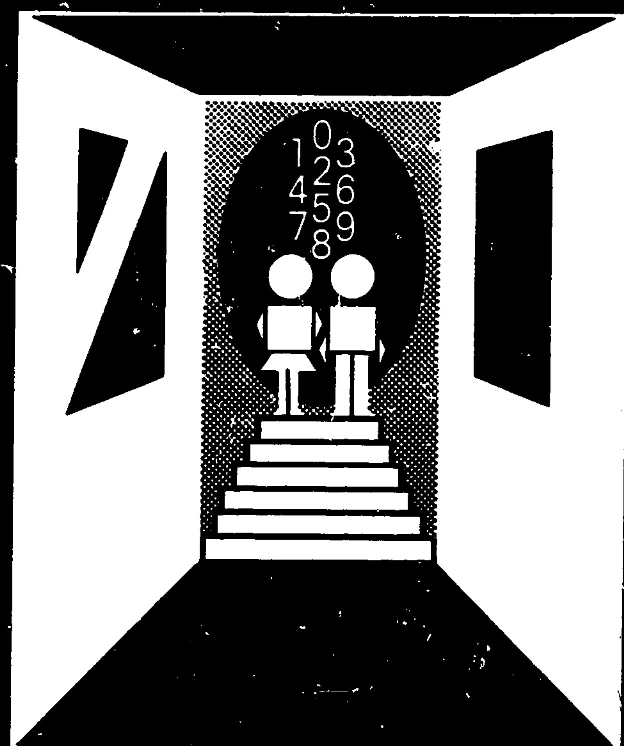

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Development Process for the Assessment Objectives

In 1969 the National Assessment of Educational Progress (NAEP) was established for the purpose of collecting information about the performance of 9-, 13-, and 17-year-old Americans in a variety of subject areas. Over the last two decades, information has been collected about performance in mathematics, writing, science, music, art, and social studies, as well as other disciplines. Assessments in mathematics were conducted during the school terms ending in 1973, 1978, and 1982, and a fourth is being held during 1986.

For each of the assessments, a process was used to collect information from educators and lay citizens to ascertain the content and trends in mathematics at that time. This information was then used to develop or revise assessment objectives. As educators' and researchers' views of mathematics have changed over the last decades, so has the assessment of mathematics. In addition, as needs have emerged for information for varying purposes—making policy decisions, understanding student competencies more clearly, determining relationships between subject areas—so too have the assessments changed. The objectives for each assessment have reflected these changes, yet they have preserved the underlying premise: that they should reflect the thinking of a wide range of individuals with an interest in mathematics. Therefore, parents and other members of the general public, as well as educators, researchers, and teacher educators, have participated in the objective-setting process.

A list of objectives for the first NAEP mathematics assessment was developed through detailed planning and study by mathematicians and mathematics educators and with reviews by panels of interested lay citizens. The resulting statement of objectives was compared to other statements of objectives that had appeared in the mathematics literature during the preceding 25 years and were found to be consistent with them. This outcome was both desired and expected since one criterion for the National Assessment objectives was that they be central to prevailing teaching efforts. A booklet containing the final set of objectives was published in 1970.

Objectives and exercises for the second assessment were developed through a series of conferences. Participants included college or university mathematics educators, mathematics classroom teachers, and interested lay citizens. An advisory board was then formed and was instrumental in organizing the final set of objectives, planning the development of exercises, selecting the final exercises, and planning subsequent reports.

Similar procedures were used to develop the third assessment's objectives and exercises. The objectives were based on the framework used for the second assessment, with some revisions that reflected content changes and trends in school mathematics. A nine-member advisory committee was instrumental in reviewing the objectives and giving guidance throughout the developmental process.

As has been the case in earlier assessments, the objectives for the 1985-86 assessment were derived through a process of review and revision. Studies were conducted to determine the needs, opinions, interests, and priorities in mathematics education. First, 25 mathematics educators and classroom teachers reviewed the objectives used for the previous assessment (1981-82). Their responses were collated by staff members and were given to the Mathematics Learning Area Committee (See Appendix A) for use in updating the list of mathematics objectives for the 1985-86 assessment. This committee was also given the NAEP Assessment Policy Committee's request that all 1985-86 assessment areas (reading, mathematics, science, and computer competence) focus on higher-level, critical-thinking skills. The draft of objectives prepared by the committee was reviewed and additional comments and suggestions were submitted by another panel of 25 mathematics educators and classroom teachers. These responses, collated by staff members, were used by the Mathematics Learning Area Committee in preparing the final list of assessment objectives.

The Description of What Is Assessed

The mathematics objectives are organized into five broad areas:

(A) Problem Solving/Reasoning

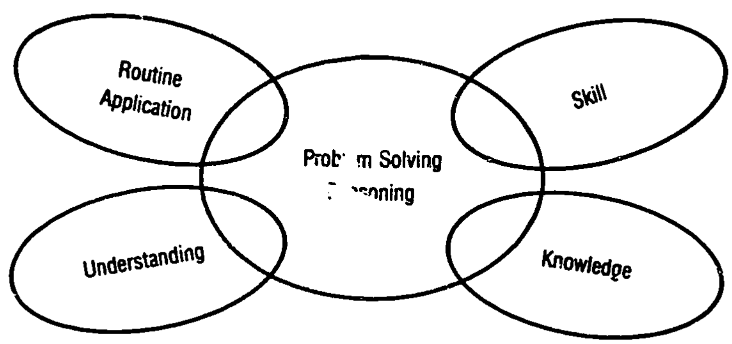
(B) Routine Application

(C) Understanding/Comprehension

(D) Skill

(E) Knowledge

Since all of the objectives involve elements of problem solving and since the Assessment Policy Committee requested an emphasis on critical thinking and problem solving, the relationships among the five process areas of the objectives can be seen in the following diagram:



The content is primarily from elementary and secondary school mathematics up to but not including the calculus. The categories of mathematics content assessed are:

(1) Fundamental Methods of Mathematics

(2) Discrete Mathematics

(3) Data Organization and Interpretation

(4) Measurement

(5) Geometry

(6) Relations, Functions, and Algebraic Expressions

(7) Numbers and Operations

The Objectives Framework for the Fourth Mathematics Assessment in Figure 1 shows the relationship between the process areas of learning and the mathematics content. The process and content areas are described in the following sections of the booklet.

Figure 1

Objectives Framework for the Fourth Assessment

CONTENT

PROCESSES	(1) Mathematical Methods	(2) Discrete Mathematics	(3) Data Organization and Interpretation	(4) Measurement	(5) Geometry	(6) Relations, Functions, etc.	(7) Numbers and Operations
(A) Problem Solving/ Reasoning							
(B) Routine Application							
(C) Understanding/ Comprehension							
(D) Skill							
(E) Knowledge							

Process Areas

(A) *Process Area: Problem Solving/Reasoning*

This category of exercises is intended to assess higher order thinking skills. Therefore, the exercises require processes that are intellectually more complex than the application of skills or the understanding of a single concept.

In the area of problem solving, the exercises require such processes as identifying and using a problem-solving strategy, screening relevant from irrelevant information, formulating a problem or selecting a model of a problem situation, determining what information would be needed to solve a problem, or organizing given information to represent the problem. The category also includes such processes as formulating generalizations or testing their validity, recognizing patterns and describing or symbolizing the relationships, or informally

making inferences. In contrast to exercises in the categories of routine problem solving and understanding, an exercise in this category might ask the student to identify *all* needed information to solve a non-routine problem.

(B) *Process Area: Routine Application*

Routine mathematical application refers to the use of mathematical knowledge, skill, and understanding in solving problems that are routine in the sense of familiarity—similar problems would have been studied either in the course of instruction or in a textbook assignment. The student is thus presumed to have had experience in solving comparable problems, and transfer to new situations is minimal. That is, while the student is not told how to solve the problem, the stimulus is such that selection of an appropriate procedure is almost automatic. Exercises assessing routine application do not vary much from textbook problems. An exercise might require, for example, the solution of a standard problem on proportion, the demonstration that two geometric figures are congruent, or an estimate of the amount of carpet needed for a room.

(C) *Process Area: Understanding/Comprehension*

Mathematical understanding, or comprehension, refers to the interpretation and elaboration of underlying concepts, assumptions, relationships, and the like. These underpinnings may be as elementary as the concept of a fraction or as sophisticated as the concept of a deductive system. Understanding does not rely on memory alone, but also includes the association of ideas and the perception of relationships.

Exercises assessing understanding/comprehension may require the student to identify an example (or something that is not an example) of a concept, to recognize when a particular technique may (or may not) be helpful, to give an explanation, or to translate from one mode of expression to another. For example, a student may be given partial information and be asked to identify the additional information needed in order to solve a routine problem.

(D) *Process Area: Skill*

Mathematical skill refers to straightforward, routine manipulation and relies on standard procedures that lead directly to answers. Exercises assessing mathematical skill assume that the required procedure has been learned and practiced. They do not require the student to decide which procedure to use or to apply the procedure to a new situation. Such exercises aim at measuring proficiency in carrying out a procedure rather than the understanding of how or why it works.

Mathematical skill is assessed by exercises that require the performance of specific tasks such as making measurements, multiplying two fractions, performing mental computations, graphing a linear equation, or reading a table.

(E) *Process Area: Knowledge*

Mathematical knowledge refers to both the recall and recognition of mathematical content as expressed in words, symbols, or figures. Mathematical knowledge as described for this assessment relies, for the most part, on memory; it does not ordinarily require any more complex mental processes.

Exercises that assess mathematical knowledge require that a student recall or recognize one or more items of information. Exercises involving recall might ask for a multiplication fact, such as the product of five and two, or for the statement of a mathematical relation such as the law of cosines. An exercise involving recognition might present several symbols and ask which symbol means "parallel."

The distribution of exercises by age group and process area is shown in Figure 2.

Figure 2

Approximate Percentage Distribution of Exercises by Age and Process*

	Processes	Age 9	Age 13	Age 17
(A)	Problem Solving/Reasoning	17	18	19
(B)	Routine Applications	12	15	15
(C)	Understanding/ Comprehension	19	25	28
(D)	Skill	40	32	29
(E)	Knowledge	12	11	10

*Percents may not total 100 due to rounding.

Content Areas

(1) *Content Area: Mathematical Methods*

Exercises in this content category assess an understanding of the tools of mathematics itself, those processes that are central to the extension and development of mathematics and to its use. These methods cut across all the content areas of mathematics and help to verify the various subject matter components. These processes are highly

specific to, though not always unique to, the nature of mathematics and are essential to mathematical applications in the solution of problems, both within mathematics and in other disciplines. They are assessed as objects of study and goals of instruction. Included are concepts of deductive and inductive proof, logic, models, structure and system, routine procedures, problem- solving strategies and empirical induction.

(2) *Content Area: Discrete Mathematics*

This content area reflects an increasing awareness in the school mathematics curriculum of the widening role of probability, permutations and combinations, and linear algebra in modeling applications that occur in many different subject areas. These exercises assess predictions of outcomes, mathematical expectation, permutations and combinations, applications of sums and products of matrices, and solutions of matrix equations.

(3) *Content Area: Data Organization and Interpretation*

This content area was made into a separate category in this assessment because of its emphasis in the school mathematics curriculum and its increased usage in other disciplines. The exercises assess organizing, analyzing, and interpreting data including determining measurement of central tendency and of spread.

(4) *Content Area: Measurement*

The exercises in this content area assess developmental concepts of measurement, equivalent measurements, selection and application of appropriate size and type of units, instrument reading, and precision and accuracy. Measurements and applications of measurements of length, time, temperature, mass/weight, area, volume, capacity, and angles are included as well as measurement applications to circles, scale drawings, and money.

(5) *Content Area: Geometry*

Exercises in this category assess properties and relations of geometric figures such as parallel lines, perpendicular lines, similar polygons, congruent figures, vertical angles, angles of a triangle, and measures of central angles of a circle as well as the Pythagorean relation. Relations established by formal proofs such as various loci of points, angles inscribed in semicircles, and special right triangles are included. Also assessed are lines of symmetry and images of figures under flips, turns, and slides, and other spatial relations in two and three dimensions such as intersections of planes and solids.

(6) *Content Area: Relations, Functions, and Algebraic Expressions*

This content area is broad in scope. Exercises assess the use of variables in expressions of relationships; translations from words to symbols; and use of variables to represent properties of operations, number theory concepts, and properties of equality and inequality. Determining solutions of equations and inequalities including systems of equations and quadratic equations is assessed. Generalization of patterns; evaluation and interpretation of functions and formulas; plotting of graphs in rectangular and other coordinate systems, and exponential and trigonometric functions are also included in this content area.

(7) *Content Area: Numbers and Operations*

In this content area concepts of numeration and number are assessed for whole numbers, common fractions, decimal fractions, integers, and percents. Operations with these numbers, including mental computation, are assessed as is estimation of computation. Number properties and relationships such as number patterns and ratio and proportion are assessed in this content area.

An outline of these content areas is provided in Appendix B. The distribution of exercises by content category and age group are shown in Figure 3.

Figure 3

Approximate Percentage Distribution of Exercises by Age and Content*

	Content	Age 9	Age 13	Age 17
(1)	Mathematical Methods	12	9	10
(2)	Discrete Mathematics	2	4	5
(3)	Data Organization and Interpretation	11	8	6
(4)	Measurement	19	17	13
(5)	Geometry	5	8	11
(6)	Relations, Functions, etc.	5	5	13
(7)	Numbers and Operations	47	49	43

*Percents may not total 100 due to rounding.

Problem Solving

Problem solving/reasoning is one of the categories in the process areas and has been addressed; however, problem solving, in its broadest meaning, was incorporated into all content areas of this mathematics assessment. More than just the ability to select solutions of routine word problems is to be assessed. The Mathematical Learning Area Committee prepared a brief description of skills and abilities needed in problem solving. The description is provided here as further clarification. It is not intended to be all-inclusive or a set of objectives.

Brief Description of Problem-Solving Skills and Abilities

Pre-analysis skills such as:

- the identification of mathematical problems or mathematical questions that could arise from a given description of a practical situation or a given mathematical model (graph, diagram, table, system of equations, etc.)
- the formulation of reasonable mathematical hypotheses from given information or a general description of a situation

Analysis skills such as:

- the identification of known facts, unknowns, or questions in a given problem situation
- the identification of information/data needed to solve a given problem, any extraneous information, and the interpretation of technical terms in the problem
- the recognition of problems in which the underlying mathematical processes are the same as a given problem, but the context is different

Ability to select viable strategies for the solution of a given problem such as:

- the identification of general strategies (appropriate drawings, graphs, tables, simpler problems, patterns, "guess and check," "working backwards," etc.) that may help in the solution of a problem
- the identification of specific procedures (sequences of steps and/or operations) that will lead to a solution of a problem

- the use of estimation to predict a reasonable solution for a given problem

Ability to interpret the solution of a problem such as:

- the recognition and/or verification of a sensible solution for a given problem
- the identification of new relationships, or the prediction of possible consequences based upon the solution of a given problem or problems

Attitudes

NAEP has assessed attitudes toward mathematics beginning with the second assessment in 1977-78. Five categories of attitudinal measures were developed and have been used: (1) mathematics in school, (2) mathematics and oneself, (3) mathematics and society, (4) mathematics as a discipline, and (5) attitudes toward computers.

Calculator

Because of the increasing availability and popularity of calculators, NAEP has gathered information about their use by students beginning with the 1977-78 mathematics assessment.

The assessment includes five categories of exercises. They are: (1) routine computation, (2) more difficult computations, (3) understanding computers, (4) exploration, and (5) application or problem solving. Some calculator activities, such as understanding and exploration, are more appropriate for instructional use in the classroom and are not emphasized in the assessment. Thus, of the five categories, computation, nonroutine computation, and application are measured in the greatest depth. Additional information on these categories can be found in Appendix C.

A minimal amount of instruction on the use of the calculator is given prior to the exercises. Many of the exercises are repeated, but without the use of calculators, in other parts of the assessment for the same age group to permit comparisons of performance with and without the calculator.

Background information is gathered from each student as to experience with calculators. The questions include how often the student has used a calculator, if the student's family owns one, in what courses the student has used a calculator, and what experiences with calculators the student has had outside of school.

Questions to be Addressed by the Assessment

In developing and selecting exercises for the assessment, care was taken to ensure an appropriate balance of emphasis on both the content and process dimensions. NAEP previously organized questions according to the process and attitude dimensions and included the topics of the content dimension. The questions were based on priorities of the mathematicians, mathematics educators (including teachers), educational administrators, and lay people involved in the developmental process of the assessment. The following questions incorporate the revisions made by the Mathematics Learning Area Committee for the 1985-86 assessment and provide an overview of the assessment.

A. Mathematical problem solving/reasoning

1. How well can students analyze a situation and formulate a mathematical problem?
2. How well can students screen relevant from irrelevant information in a problem situation?
3. How well can students organize given information in a problem situation into a usable form (table, graph, chart, diagram, or algebraic expressions, etc.)?
4. How well can students identify strategies and procedures that may help in solving a problem?
5. How well can students carry out procedures selected for solving a given problem?
6. How well can the students check for reasonableness of an answer that resulted from a procedure used to solve a problem?

B. Routine mathematical application

1. How well can students solve a routine textbook problem?
2. How well can students select a procedure to solve a routine application?
3. How well can students apply problem-solving strategies?
4. How well can students estimate an answer to an application problem?
5. How well-can students interpret data and draw conclusions?
6. How well can students use mathematics, including logic, in reasoning and making judgments?

7. How well can students use a calculator to solve an application problem?

C. Mathematical understanding/comprehension

1. How well can students translate a verbal statement into symbols or a figure, and vice versa?
2. How well do students understand mathematical concepts and principles?
3. How well can students see relations between the information and questions in a problem situation and simultaneously between those two elements and the concepts, principles, and processes previously acquired?
4. How well can students select and draw on resources such as paper and pencil, computer, calculator, or estimation?

D. Mathematical skill

1. How well can students perform paper-and-pencil computations, including computations with whole numbers, integers, fractions, decimals, percents, and ratios and proportions?
2. How well can students perform algebraic manipulations?
3. How well can students perform geometric manipulations such as constructions and spatial visualizations?
4. How well can students make measurements?
5. How well can students read graphs and tables?
6. How well can students compute statistics, probabilities, permutations or combinations?
7. How well can students perform mental computations, including computation with whole numbers, fractions, decimals, and percents?
8. How well can students estimate measurements and answers to computations?
9. How well can students perform computations involving whole numbers, decimals, fractions, and percents using calculators?
10. How well can students read flow charts or simple computer programs?

E. Mathematical knowledge

1. How well can students recall and recognize facts, definitions, and symbols?

F. Attitudes

1. How do students feel about the mathematics they encounter in school?
2. How do students feel about the various activities in mathematics classes?
3. How do students feel about their personal experience with mathematics?
4. What are students' beliefs about the nature of mathematics as a discipline?
5. What are students' beliefs about the value of mathematics to society?
6. What are students' beliefs about computers?

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Content Outline

The following list is an outline of content for which the test questions were written. Not all the subtopics are tested at every age level. Students are not tested on the meanings of these terms. Questions are stated in the students' language and at the students' level of maturity.

1. Fundamental Methods of Mathematics
   a. Models
   b. Problem-solving strategies (heuristics)
   c. Informal induction
   d. Informal deduction
   e. Routine procedures (algorithms)
   f. Axiomatic systems
   g. Logic
2. Discrete Mathematics
   a. Probability (simple, compound, and independent events; odds; mathematical expectation)
   b. Permutations and combinations
   c. Matrices (sums and products of matrices, matrix equations)
3. Data Organization and Interpretation
   a. Organizing, displaying, and interpreting information (tallies, tables, charts, and graphs)
   b. Measures of central tendency (mean, median, mode)
   c. Measures of spread and position (range)
   d. Sampling and polling (effects of sampling, random samples, opinion polls)
4. Measurement
   a. Concepts of measurement (nonstandard units, iteration of a unit model, additive property)
   b. Standard units of measure (appropriate size and type of unit, conversions within a system, estimation of measurements)
   c. Measuring instruments (reading of instruments, precision/accuracy of measurement)
   d. Applications of measurement (time, money, perimeter, circle, scale drawings, etc.)

5. Geometry
   a. Geometric figures and their properties (polygons, solids, circles, angles, curves, lines, line segments, rays)
   b. Relations established by definition (parallelism, perpendicularity, similarity, congruence)
   c. Relationships established by theorem (informally presented: angles opposite congruent sides of triangle, vertical angles, sum of angles of triangle, Pythagorean relation, measures of central angles of a circle; formal proof: loci, parallel lines, parallelograms, angle sum for n-gon, inscribed quadrilaterals, angles inscribed in a semi-circle, special right triangles, constructions)
   d. Motion geometry (informal: lines of symmetry; flips, turns, and slides)
   e. Coordinate systems applied to geometry
   f. Spatial visualization (orientation in 3-dimensional space, decomposition of irregular shapes into familiar shapes, intersections of point sets—in 2-dimensional and 3-dimensional spaces)
6. Relations, Functions, and Algebraic Expressions
   a. Use of variables (translation from verbal to symbolic; representations of properties of operations, equalities, and inequalities; equivalent equations and inequalities; simplification of algebraic expressions)
   b. Simple relations and functions (generalization of patterns, direct and indirect variation, evaluation of formulas and functions, domain and range of functions)
   c. Solving equations and inequalities (simple equations, inequalities, systems of equations, quadratic equations)
   d. Coordinate systems (rectangular, polar, and 3-dimensional)
   e. Functions (pre-calculus: properties of functions and inverse of a function; graphs of functions and relations: algebraic, exponential, and logarithmic)
   f. Trigonometry (identities, radian measure, solution of triangles, equations, graphs of functions)

7. Numbers and Operations
   a. Numeration (rounding, whole numbers, fractions, decimals, percents, integers, scientific notation)
   b. Number concepts (order, equivalence, number line, whole number, fraction, decimal, percent, integer, changing a representation of a number from one form to another)
   c. Operations (fundamental operations: concepts, whole numbers, fractions, decimals including percents, integers, combinations; inverse operations; powers and roots)
   d. Mental computation
   e. Estimation (computational and applications)
   f. Properties
   g. Relationships (number patterns, ratio and proportion, and inequaiities)

Calculator Usage

The use of the calculator has been included in the National Assessment of Educational Progress beginning with the second assessment. At that time the availability and popularity of calculators made it important for the National Assessment to collect information on their use by students. With the widespread use of inexpensive calculators, the mathematics curriculum continues to be influenced by their use.

Several conferences were organized by NAEP to discuss the place of calculators in the second assessment. Five categories of exercises for which calculators might be used in the classroom or for assessment were identified. These categories are:

1. *Routine Computations*—Computations with whole numbers, decimals, fractions, and integers that are typical of the mathematics curriculum could be included, not to replace computation as taught in the past, but to include the use of calculators as tools of mathematics.
2. *More Difficult Computations*—Students could be asked to perform more difficult computations than the routine, or computations for which algorithms are not formally taught. For example, nine-year-olds might be asked to do computations with decimals. Thirteen-year-olds might be given questions requiring conversion between fractions and decimals. Students at all ages could be given exercises requiring work with large numbers or complicated decimals that would make computation without a calculator tedious.
3. *Understanding Concepts*—Nine-year-olds might use the calculator to learn more about place value, and 13-year-olds might learn estimation for order of magnitude. Seventeen-year-olds might use the calculator to facilitate learning order of operations, extracting square roots, graphing functions, and understanding properties of functions.
4. *Exploration*—Number theory could be included. Problems involving series, summations, patterns, or divisibility are typical in this category.
5. *Applications and Problem Solving*—Routine and more difficult word problems, such as multistep problems, would be included in this category. For nine-year-olds, the use of the calculator permits some problems that require larger numbers than those to which students are accustomed. Problems for 13-year-olds

might involve percent, unit pricing, and more difficult word problems. Seventeen-year-olds might be given a variety of realistic consumer problems as well as exercises requiring the use of mathematical formulas.

The five categories could be assessed at each of the three age levels; however, practical considerations impose limitations on what can be assessed. Exercises in some categories, such as understanding concepts and exploration, are more appropriate for instructional purposes than for assessment. Therefore, categories 1, 2, and 5 are measured in the greatest depth.

Some exercises are given with and without calculators to provide data on student performance with and without the aid of the calculator.